

Index theorem for sheaves and D-modules

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Abstract

The classical Riemann-Roch-Hirzebruch theorem calculates the Euler-Poincaré index of the cohomology of a coherent sheaf on a compact complex manifold. This theorem has been generalized in many directions: to the relative case by Grothendieck, to elliptic systems on a compact real manifold by Atiyah-Singer, to \mathbb{C} -constructible sheaves by McPherson, to \mathbb{R} -constructible sheaves by Kashiwara and more recently to “elliptic pairs” (a mixture of \mathbb{R} -constructible sheaves and \mathcal{D} -modules) by Schapira-Schneiders.

In these lectures, I will try to treat all these questions in a unified way, with the tools of (microlocal) sheaf theory. The course will be adapted to the audience but I plan to treat

- Sheaves (six operations as well as microlocalization),
- a glance to \mathcal{D} -modules,
- Hochschild classes and index theorem for elliptic pairs.

References

- [Ka03] M. Kashiwara, *D-modules and Microlocal Calculus*, Translations of Mathematical Monographs, **217** American Math. Soc. (2003).
- [Sc14] P. Schapira, *An introduction to the microlocal theory of sheaves*, <http://webusers.imj-prg.fr/~pierre.schapira/lectnotes>